PROPAGATION OF A THERMAL WAVE IN A NONLINEAR ABSORBING MEDIUM

L. K. Martinson

UDC 536.24

We consider nonlinear heat transport in a medium whose thermal conductivity has a power-law temperature dependence. When the volumetric heat absorption rate in the medium is temperature dependent this process is described [1] by the quasilinear parabolic equation

$$\partial u/\partial t = \operatorname{div}(u^{\sigma} \operatorname{grad} u) - \psi(u), \ \sigma > 0. \tag{1}$$

A characteristic feature of this nonlinear transport process is the finite rate of propagation of thermal disturbances [2, 3]. This means that a disturbance from a source is propagated in the medium in the form of a thermal wave whose front moves at a finite velocity with respect to the undisturbed background. When there is a temperature-dependent volume absorption of heat in the medium ($\psi \neq 0$) a nonlinear effect of the spatial localization of a thermal disturbance can be observed when even after an infinite time the disturbance penetrates only a finite distance into the medium [4, 5].

In the present paper we investigate the localization of a thermal disturbance when the velocity of the thermal wavefront reverses its direction.

We consider the effect of an instantaneous point heat source of strength Q placed in a nonlinear medium in which volumetric heat absorption has a power-law temperature dependence ($\psi(u) = \gamma u^{\nu}$, γ = const, $\nu = 1 - \sigma$, $\sigma < 1$). In this case the problem of finding the temperature distribution in the medium reduces to the problem of solving the quasilinear equation

$$\frac{\partial u}{\partial t} = \frac{1}{x^{s-1}} \frac{\partial}{\partial x}\left(x^{s-1} u^{\sigma} \frac{\partial u}{\partial x}\right) - \gamma u^{1-\sigma}, \tag{2}$$

where s = 1, 2, and 3 for plane, axially symmetric, and centrally symmetric problems, respectively. The initial distribution $u(o, x) = u_0(x)$ in the Cauchy problem under consideration in the domain $R_+^2 = \{(t, x) : t \in [0, +\infty), x \geq 0\}$ for Eq. (2) is represented by a delta function which satisfies the condition

$$\int_0^{\infty} u_0(x) L(s) x^{s-1} dx = Q, \tag{3}$$

where

$$L(s) = \begin{cases} 2 & \text{for } s = 1, \\ 2\pi & \text{for } s = 2, \\ 4\pi & \text{for } s = 3. \end{cases}$$

Using the results of [6] we seek the solution of problem (2), (3) in the form

$$u(t, x) = \begin{cases} a(t)\left[x_+^2(t) - x^2\right]^{\alpha} & \text{for } x < x_+(t), \\ 0 & \text{for } x \geqslant x_+(t). \end{cases} \tag{4}$$

Substituting u(t, x) in the form (4) into Eq. (2), we obtain

$$\dot{a}\left[x_+^2 - x^2\right]^{\alpha} + 2\alpha a x_+ \dot{x}_+ \left[x_+^2 - x^2\right]^{\alpha-1} = 2\alpha s a^{\sigma+1}\left[x_+^2 - x^2\right]^{\alpha-1} + 4\alpha(\alpha-1) a^{\sigma+1} x^2 \left[x_+^2 - x^2\right]^{\alpha-2} - \gamma a^{1-\sigma}\left[x_+^2 - x^2\right]^{\alpha(1-\sigma)}. \tag{5}$$

A dot over a quantity in (5) denotes its time derivative. Equation (5) is satisfied identically if we set $\alpha = \sigma^{-1}$ and choose the relations $a(t)$ and $x_+(t)$ in the form

$$a(t) = \left[\frac{\sigma}{2(2 + s\sigma)t}\right]^{\frac{1}{\sigma}}; \tag{6}$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 36–39, July–August, 1979. Original article submitted June 29, 1978.

0021-8944/79/2004-0419$07.50

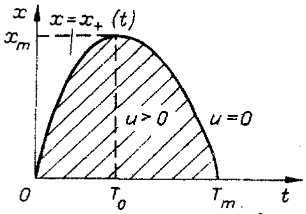

Fig. 1

$$x_+^2(t) = Ct^{\frac{2}{2+s\sigma}} - \gamma \frac{(2+s\sigma)^2}{1+s\sigma} t^2. \tag{7}$$

The constant C in Eq. (7) can be determined by using the integral condition (3) in the form

$$\int_0^{x_+(t)} u(t, x) L(s) x^{s-1} dx = Q \quad \text{as} \quad t \to 0.$$

After evaluating the integral [7] and taking account of (4), (6), and (7), we obtain

$$C = \left[\frac{Q}{R(s, \sigma)}\right]^{\frac{2\sigma}{2+s\sigma}}, \quad R(s, \sigma) = \frac{1}{2} L(s) \left[\frac{\sigma}{2(2+s\sigma)}\right]^{\frac{1}{\sigma}} B\left(\frac{s}{2}, \frac{\sigma+1}{2}\right). \tag{8}$$

Thus, the final exact solution of problem (2), (3) has the form

$$u(t, x) = \begin{cases} \left\{\frac{\sigma}{2(2+s\sigma)t} \left[x_+^2(t) - x^2\right]\right\}^{\frac{1}{\sigma}} & \text{for } x < x_+(t), \\ 0 & \text{for } x \geqslant x_+(t), \end{cases} \tag{9}$$

where $x_+(t)$ is determined by Eq. (7) and the constant C is found from (8). We note that as $\gamma \to 0$ this solution goes over into the solution of the problem of a point source in a nonlinear medium without absorption [2].

We give a physical interpretation of solution (9). This solution describes a thermal wave whose front moves through the medium with a finite velocity. At any instant the position of the thermal wavefront, which separates the disturbed region where $u > 0$ from the undisturbed region where $u = 0$, is determined by the relation $x = x_+(t)$. The character of this relation is shown qualitatively in Fig. 1. We note that at the thermal wavefront the temperature and heat flux are continuous, even for $\sigma < 1$ when the wavefront is steep ($|u_+| \to \infty$ as $x \to x_+(t)$). In this case the heat flux across the wavefront vanishes because the thermal conductivity vanishes.

A characteristic feature of the problem under consideration is the fact that in the process of evolution of the thermal disturbance the thermal wavefront reverses its direction of motion. For $t < T_0$, where

$$T_0 = \left[C \frac{1+s\sigma}{\gamma(2+s\sigma)^3}\right]^{\frac{2+s\sigma}{2(1+s\sigma)}},$$

$\dot{x}_+(t) > 0$ and the sizes of the disturbed region increase with time. The velocity of the front decreases, and at $t = T_0$ the thermal wavefront stops. At this instant the disturbed region reaches its maximum size $x_m = x_+(T_0)$. Then the velocity of the front reverses, and for $t > T_0$ a cooling wave is propagated in the medium ($\dot{x}_+(t) < 0$). For $t = T_m$, where

$$T_m = T_0 (2+s\sigma)^{\frac{2+s\sigma}{2(1+s\sigma)^2}}$$

the disturbed region contracts to a point, and $u = 0$ everywhere in $R_+^2 \cap \{t > T_m\}$. In other words, in the problem under consideration the thermal disturbance from an instantaneous point source exists for a finite time T_m. This result is unusual from the point of view of the linear theory of heat conduction, but it agrees with the conclusions in [8] where it was shown that for $\nu < 1$ in the Cauchy problem for Eq. (1) a value of $T_m < +\infty$ always exists such that $u(t, x) = 0$ for all $t \geq T_m$.

In conclusion we note that Eq. (1) describes a broad class of transport phenomena (heat conduction, diffusion, filtration, etc.). Therefore the results obtained can be interpreted within the framework of other physical models.

LITERATURE CITED

1. A. N. Tikhonov and A. A. Samarskii, Equations of Mathematical Physics, Macmillan (1963).
2. Ya. B. Zel'dovich and A. S. Kompaneets, "On the theory of propagation of heat for a temperature-dependent thermal conductivity," in: Collection Dedicated to the Seventieth Birthday of A. F. Ioffe [in Russian], Akad. Nauk SSSR, Moscow (1950).
3. O. A. Oleinik, A. S. Kalashnikov, and Wei-Liang Chow, "The Cauchy problem and boundary-value problems for an equation of the unsteady filtration type," Izv. Akad. Nauk SSSR, Ser. Mat., 22, No. 5 (1958).
4. L. K. Martinson and K. B. Pavlov, "On the problem of the spatial localization of thermal disturbances in the theory of nonlinear heat conduction," Zh. Vychisl. Mat. Mat. Fiz., 12, No. 4 (1972).
5. S. I. Golaido, L. K. Martinson and K. B. Pavlov, "Unsteady problems of nonlinear heat conduction with volumetric heat absorption," Zh. Vychisl. Mat. Mat. Fiz., 13, No. 5 (1973).
6. R. Kershner, "On certain properties of the generalized solutions of quasilinear degenerate parabolic equations," Author's Abstract of Thesis, Moscow State University (1976).
7. I. S. Gradshtein and I. M. Ryzhik, Tables of Integrals, Series, and Products, Academic Press (1965).
8. A. S. Kalashnikov, "On the nature of the propagation of disturbances in problems of nonlinear heat conduction with absorption," Zh. Vychisl. Mat. Mat. Fiz., 14, No. 4 (1974).

CONVECTIVE MASS TRANSFER IN A PERIODIC ARRAY OF SPHERES

Yu. P. Gupalo, A. D. Polyanin,
Yu. S. Ryazantsev, and Yu. A. Sergeev

UDC 532.72

In problems of convective diffusion in a system of reacting particles at high Peclet numbers, the structure of singular streamlines which begin and end on particle surfaces plays an important role [1–3]. The flow involves chains of particles in which mass transfer is greatly retarded by the interaction of diffusion wakes and boundary layers of particles belonging to the chains. Taking account of the interaction of diffusion wakes and boundary layers of particles, and assuming that the ratio of the lattice period b to the radius a of a sphere satisfies the inequality $b/a \gg Pe^{1/3}$, where Pe is the Peclet number of a single sphere, Voskanyan et al. [4] performed calculations for a system of spheres of equal radii at the nodes of a widely spaced cubic lattice. Under these assumptions the original problem could be reduced to a self-similar problem of the diffusion of matter with a constant concentration flowing past an isolated sphere [5]. In the present paper we consider mass transfer of a concentrated ordered system of reacting solid spheres when $b/a \ll Pe^{1/3}$.

We consider steady convective diffusion in the laminar flow of a viscous incompressible liquid filtering through a system of reacting spheres of equal radii at the nodes of a cubic lattice. We assume that the liquid filters through the spaces between the spheres with an average velocity U which is parallel to one axis of the lattice, and that the Reynolds number $Re = aU/\nu$, where ν is the kinematic viscosity of the liquid, is small. Then the velocity field of the liquid in the lattice can be determined within the framework of the cell model [6, 7], or when $b/a \gg 1$, by the concentrated-force model [4, 8]. Henceforth we assume that the position of a fixed sphere in the lattice is given by a set of three integers, and the distance along the stream axis is given by the value of the parameter $k = 1, 2, \ldots$.

Using a system of spherical coordinates with its origin at the center of an arbitrary sphere, the stream function near the surface of a sphere can be written in the form

$$\psi = (3/4)UA(n)(r-a)^2 \sin^2\theta, \quad \lim_{n\to 0} A(n) = 1,$$

where n is the number of spheres per unit volume. The specific expression for A(n) can be determined, in particular, from [4, 6–8].

The concentration distribution in the flow is determined by solving the steady-state convective diffusion equation

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 39–41, July–August, 1979. Original article submitted June 26, 1978.